Math 1B – Calculus – Fair Game for Test 3 – Spring '10

1. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

   a. $\int_1^\infty \frac{1}{(2x-1)^2}dx$ b. $\int_0^\infty xe^{-x^2}dx$ c. $\int_{-\infty}^\infty \frac{x^4}{64+x^6}dx$

   d. $\int_{-\infty}^\infty \frac{x^2}{64+x^6}dx$ e. $\int_0^\infty \frac{x}{4+x^2}dx$ f. $\int_0^\infty \frac{x}{4+x^3}dx$

   g. $\int_0^\infty \frac{1}{x^2\sqrt{x^2+1}}dx$ h. $\int_1^\infty \frac{1}{x^2\sqrt{x^2-1}}dx$ i. $\int_1^\infty \frac{1}{(x-1)^2\sqrt{x}}dx$

2. Use the comparison theorem to show that the integral is either convergent or divergent.

   a. $\int_{100}^\infty \frac{1-e^{-x}}{x^2}dx$ b. $\int_{100}^\infty \frac{1-e^{-x}}{x}dx$ c. $\int_3^\infty \frac{\sin^2 x}{\sqrt{x^3}-3}dx$

3. Consider the integral $\int_0^\infty x^n e^{-x}dx$.
   a. Evaluate the integral for $n = 0,1, 2,$ and 3.
   b. Make a guess about the value of the integral in terms of n.
   c. Prove your guess is right by using mathematical induction.

4. Find the energy that is needed to propel a rocket of mass m from the surface of a planet out of the gravitational field of that planet (to infinity). Assume the planet has mass M and radius R. Use Newton's Law of Gravitation for the gravitational force: $F=\frac{GmM}{r^2}$, where G is the gravitational constant and r is the distance of the rocket to the center of the planet.

5. If $f(t)$ is continuous for $t \geq 0$, the Laplace transform of f is the function F defined by
$$F(s)=\int_0^\infty f(t)e^{-st}dt$$
   and the domain of F is the set consisting of all numbers s for which the integral converges. Find the Laplace transforms of the following functions:
   a. $f(t) = 2$
   b. $f(t) = t^2$

6. Determine the value of a so that $\int_0^a \frac{t}{t^2+1}dt = e^{1000}$

7. If the integrals $\int_a^\infty f(t)dt$ and $\int_{-\infty}^a f(t)dt$ are convergent then it's possible to define $\int_{-\infty}^\infty f(t)dt=\int_{-\infty}^a f(t)dt+\int_a^\infty f(t)dt$. With this in mind, is possible to define $\int_{-\infty}^\infty \frac{t}{t^2+1}dt$?

8. Use the arc length formula to find the circumference of the unit circle.

9. Find the length of the curve:
   a. $y = \dfrac{x}{14} + \dfrac{7}{\sqrt{x}}$ from $x = 1$ to $x = 7$.
   b. $y = 1 + \dfrac{1}{3}\sinh 3x$ from $x = 0$ to $x = 1$.
   c. $y = \sqrt{x - x^2} + \arcsin\left(\sqrt{x}\right)$ (over the entire domain)
   d. $y = \int_0^x \sqrt{1 + \cos t}\, dt$ from $x = 0$ to $x = \pi$.

10. A steady wind blows a kite due west. The kite's height above ground from horizon position $x = 0$ to $x = 80$ meters is given by $y = 150 - 0.025(x - 50)^2$. Find the distance traveled by the hawk.

11. Find the surface are generated by rotating about the x–axis the parabola $x = y + y^2$ for $1 \le y \le 2$.

12. Find the surface area generated by rotating about the y–axis the parabola $y = \dfrac{1}{2}x^2 - \ln x$ for $1 \le x \le 3$.

13. A group of engineers is building a parabolic satellite dish whose shape will be formed by rotating the curve $y = ax^2$ about the y-axis. If the dish is to have a 10-ft diameter and a maximum depth of 2 ft, find the value of and the surface area of the dish.

14. Show that the surface area of a zone of a sphere that lies between two parallel planes is $S = \pi dh$, where d is the diameter of the sphere and h is the distance between the planes. (Notice that S depends only on the distance between the planes and not on their location, provided that both planes intersect the sphere.)

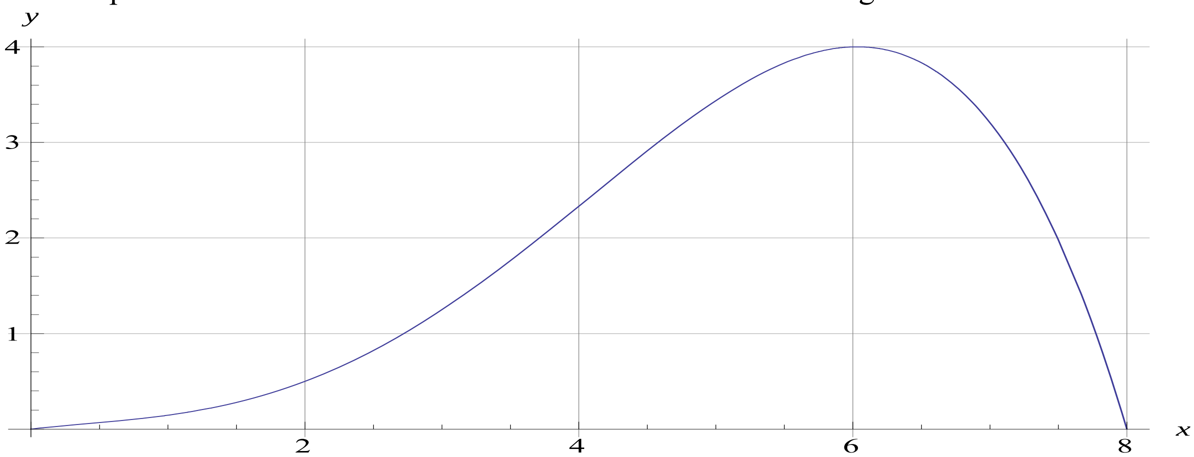
15. If the infinite curve $y = e^{-x}$, $x \ge 0$, is rotated about the x-axis, find the area of the resulting surface.

16. Find the hydrostatic force on the lune form by the region inside unit circle, $x^2 + y^2 = 1$ and outside the circle $(x + 1)^2 + y^2 = 2$ submerged underneath a water level $y = 7$.

17. Find the moments and center of mass of the system of objects that have masses 5, 8 and 13 at the points $(-1,2)$, $(3,1)$ and $(4,-1)$, respectively.

18. Find the center of mass of the crescent plate inside unit circle, $x^2 + y^2 = 1$ and outside the circle $(x + 1)^2 + y^2 = 2$.

19. Use Simpson's method with $n = 4$ to estimate the centroid of the region shown:



20. Let be the probability density function for the time it takes you to drive to school in the morning, where is measured in minutes. Express the following probabilities as integrals.
(a) The probability that you drive to school in less than 15 minutes
(b) The probability that it takes you more than half an hour to get to school

21. Let $f(x) = kx^2(1-x)$ if $0 \le x \le 1$ and $f(x) = 0$ if $x < 0$ or $x > 1$.
   a. For what value of k is $f(x)$ a probability density function?
   b. For that value of k, find $P(X \geq 0)$
   c. Find the mean.

22. According to the National Health Survey, the heights of adult males in the United States are normally distributed with mean 69.0 inches and standard deviation 2.8 inches.
   a. What is the probability that an adult male chosen at random is between 65 and 73 inches tall?
   b. What percentage of the adult male population is more than 6 feet tall?

23. Consider $f(x) = \dfrac{C}{x^2 - 2x + 2}$
   a. For what value of C is $f(x)$ a probability density function?
   b. What is the mean value of random variable with probability density function $f(x)$?

24. Show that the probability density function $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$ for a normally distributed random variable has inflection points at $x = \mu \pm \sigma$.